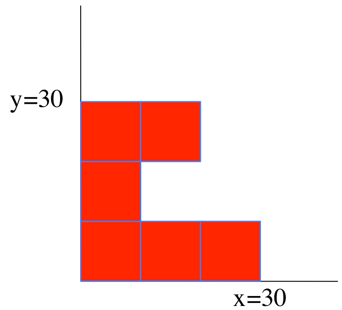
Problem 9.38

Compute the center of mass of the object pictured. Each square has mass "m."



In a two-dimensional setting, and from the perspective of mechanics, what a *center of mass* calculation is asking you to do is the following: Start at one axis and proceed out from that axis until you run into mass. Take the amount of mass you find there and multiply by how far you are from the axis (note that if it is the y-axis you are moving from, the x-coordinate of the mass will tell you how far you are from the y-axis). Do this for all the masses in the system, add those weighted position ("mx" quantities) and divide by the total mass in the system. Do that for both axes and you will have the two-dimensional coordinate for the system's *center of mass*.

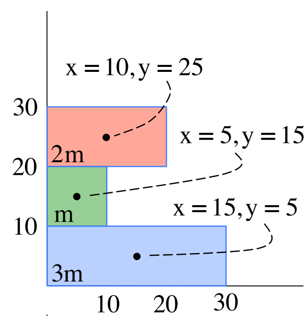
In this case, you are working with an extended object, which means you might be tempted to use

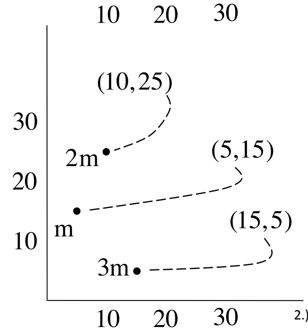
$$x_{cm} = \frac{\int x\,dm}{m_t}$$

You don't have to do that, though. Instead, you can be clever.

Consider the three rectangles defined in the sketch to the right. Each has some amount of mass, and that mass can be assumed to be compressed into a point at the rectangle's *center of mass* (this, I might add, is easy to identify just by looking). With that simplification, the problem ends up looking like the second graph shown, and we can write out the math as shown on the next page.

Minor note: Because the coordinate units are "meters," symbol "m," and the masses are in terms of "m's," we are going to have a notational problem if I include units in the calculation. As such, I will only put in units at the end.





$$y_{cm} = \frac{\sum m_i y_i}{m_{total}}$$

$$= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(3\cancel{m})(5.00) + (1\cancel{m})(15.0) + (2\cancel{m})(25.0)}{(3\cancel{m}) + (\cancel{m}) + (2\cancel{m})}$$

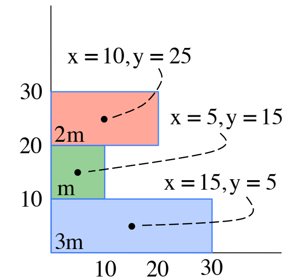
$$= 13.3 \text{ meters}$$

$$x_{cm} = \frac{\sum m_i x_i}{m_{total}}$$

$$= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(3\cancel{m})(15.0) + (1\cancel{m})(5.00) + (2\cancel{m})(10.0)}{(3\cancel{m}) + (\cancel{m}) + (2\cancel{m})}$$

$$= 11.7 \text{ meters}$$



So:

$$\vec{r}_{cm} = (11.7 \text{ meters})\hat{i} + (13.3 \text{ meters})\hat{j}$$

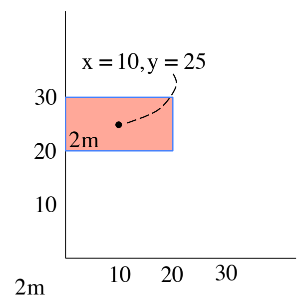
THIS IS EXTRA: To justify the eyeballed assertion that the center of mass for one of the rectangles, say the top one, was as predicted by "eyeballing it," you could do the calculation formally. Specifically (again, for the top one):

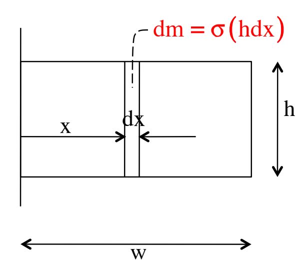
Assume an area density function of:

$$\sigma = \frac{M}{A} = \frac{M}{hw} = \frac{dm}{dA} = \frac{dm}{hdx}$$

$$\Rightarrow \quad dm = \sigma(hdx)$$

The x-coordinate of the center of mass is, which we are expecting to be at 10.0 in our situation, is calculated on the next page:





$$x_{cm} = \frac{\int_{x=0}^{w=20\,m} x\,dm}{M}$$

$$= \frac{\int_{x=0}^{w=20} x\left(\sigma h dx\right)}{M}$$

$$= \frac{\int_{x=0}^{w=20} x\left(\left(\frac{M}{hw}\right) h dx\right)}{M}$$

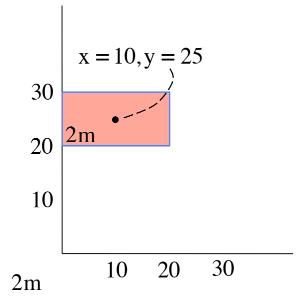
$$= \frac{1}{w}\int_{x=0}^{w=20} x\,dx$$

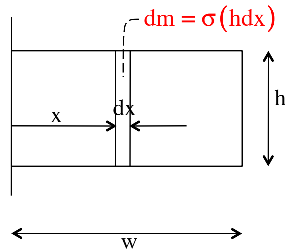
$$= \frac{1}{w}\left(\frac{x^2}{2}\right)\Bigg|_{x=0}^{w=20}$$

$$= \frac{1}{w}\left(\frac{w^2}{2}\right)$$

$$= \frac{w}{2} = \frac{20\text{ m}}{2} = 10\text{ m}$$

Apparently, we were good!





4.)